CHAPTER 3

NUMBER LINE

3.1 NUMBER LINE

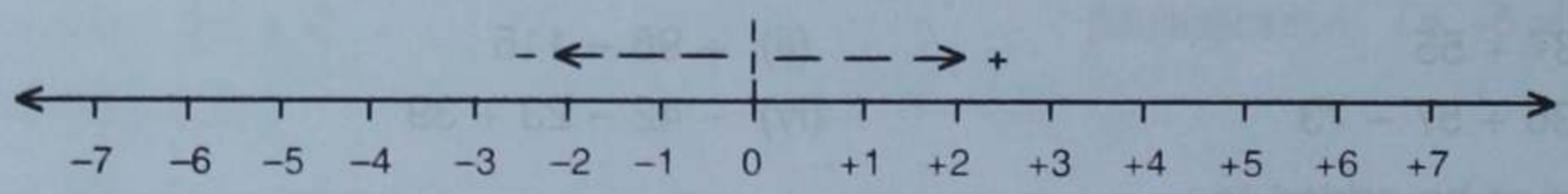
A number line can be used to represent all types of real numbers : –23, 0, 8, $\frac{3}{5}$, $2\frac{1}{5}$, $\sqrt{2}$, $\sqrt{5}$, etc. Here –23, 0 and 8 are integers and $\frac{3}{5}$, $2\frac{1}{5}$, $\sqrt{2}$ and $\sqrt{5}$ are non-integers. **But in the current chapter we shall be dealing with the number line representing integers only.**

Steps for drawing a number line :

1. Draw a straight line of any suitable length.
2. Mark points on the drawn line to divide it into the required number of equal parts.
3. Mark the centre point of the drawn straight line as zero.
4. Starting from zero, and on the right hand side of it mark the positive integers +1, +2, +3, etc., at the points marked in step 2.

   Similarly, starting from zero, on the left hand side of it mark the negative integers –1, –2, –3, etc., at the points marked in step 2.

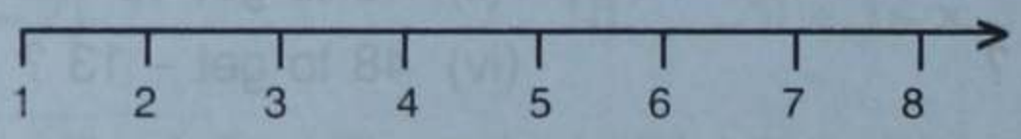
   The line so obtained will be a number line of the form shown below :



Arrow-heads at the two ends of the number line show that the line as well as the integers continue up to infinity on both the positive and the negative sides.

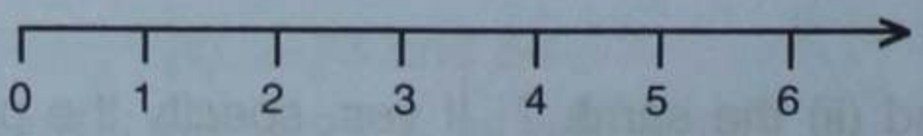
3.2 NUMBER LINES FOR NATURAL NUMBERS, WHOLE NUMBERS AND INTEGERS

1. Natural Numbers : A number line starting from 1 (one) and marked 2, 3, 4, 5, ... at equal distances on the right hand side of 1 is called ***a number line representing the natural numbers*** (as shown below) :



The arrow-head on the right side shows that the natural numbers continue up to infinity.

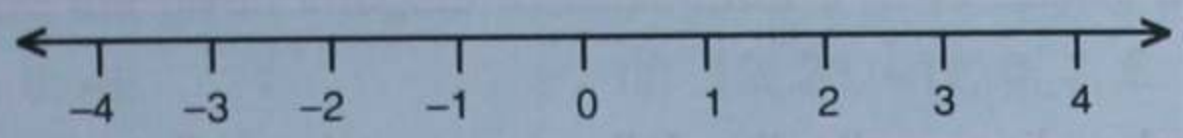
2. Whole Numbers : A number line starting from 0 (zero) and marked 1, 2, 3, 4, ... at equal distances on the right hand side of 0 is called ***a number line representing the whole numbers*** (as shown below) :



The arrow-head on the right side shows that the whole numbers continue up to infinity.

3. Integers : Since integers = { ..., – 3, – 2, – 1, 0, 1, 2, 3, 4, ... }, a ***number line with zero* (0)** marked any where on it, with positive numbers **1, 2, 3, ...** marked ***on the***

***right hand side of* 0** at equal distances and negative numbers **–1, – 2, –3, ...** marked ***on the left hand side of* 0** (zero) at the same equal distances, is said to represent **integers** (as shown below) :



Arrow-heads on the two sides show that the integers continue up to infinity on the positive side as well as on the negative side.

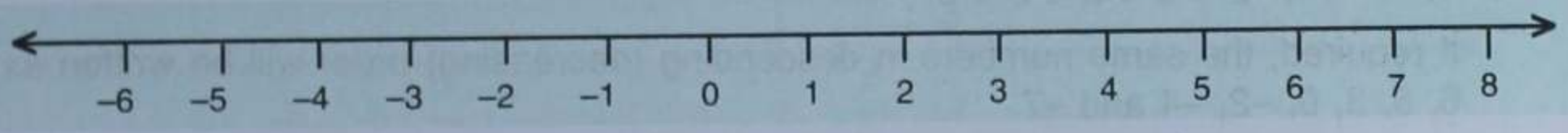
1. Integers = {....., –4, –3, –2, –1, 0, 1, 2, 3, 4, 5, ...}
2. Natural numbers = {1, 2, 3, 4, 5, 6, ...}
3. Whole numbers = {0, 1, 2, 3, 4, 5, ...}
4. Whether the number line is drawn for integers, natural numbers or whole numbers, the distance between any two consecutive numbers is always the same.

3.3 USING A NUMBER LINE TO COMPARE NUMBERS

Out of any two numbers, marked on a number line, *the* ***number*** which is

(i) ***to the right*** *is* ***greater***

(ii) ***to the left*** *is* ***smaller***.



Considering the number line drawn above :

(i) 6 is **greater** than 2 because it is **to the right** of 2

(ii) – 2 is **greater** than – 5 because it is **to the right** of – 5

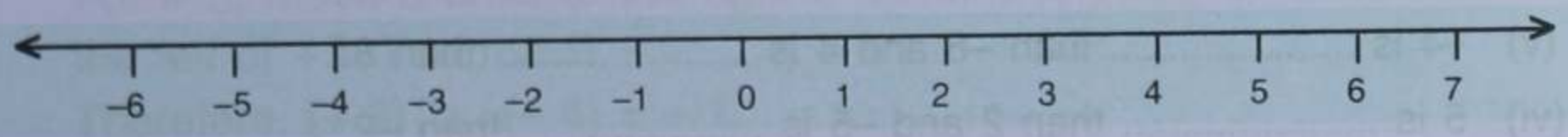
(iii) 3 is **greater** than – 1 because it is **to the right** of – 1

(iv) – 6 is **smaller** than – 2 because it is **to the left** of – 2

(v) – 4 is **smaller** than 1 because it is **to the left** of 1 and so on.

Thus, each number on a number line is always greater than each and every number to its left. Similarly, *each number on a number line is always smaller than each and every number to its right.*

For the following number line :



(i) 7 is greater than all numbers to its left
i.e. 7 is greater than each of 6, 5, 4, 3, 2, 1, 0, –1, –2, –3, *etc.*

(ii) –6 is smaller than all numbers to its right
i.e. –6 is smaller than each of –5, –4, –3, –2, –1, 0, 1, 2, *etc.*

Also, (i) *Every positive number is greater than every negative number.*

(ii) *Zero is smaller than every positive number but greater than every negative number.*

(iii) *The greater the number, the smaller is its opposite.*
viz. 8 is greater than 5 but – 8 is less than – 5
Similarly, $-9 > -15 \Rightarrow 9 < 15$ *and so on*

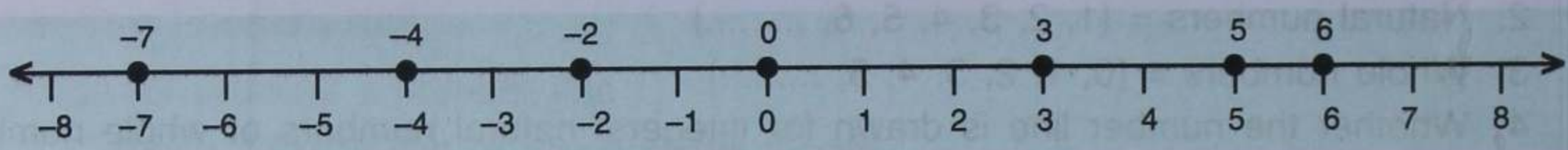
(iv) *The smaller the number, the greater is its opposite.*
viz. 6 is smaller than 7 but – 6 is greater than – 7
Similarly, $-8 < -5 \Rightarrow 8 > 5$, and so on.

Example 1 :

Using a number line, write the following numbers (integers) in ascending order of value : 3, –2, 5, 0, –7, 6 and –4.

Solution :

Draw a suitable number line and mark on it the given numbers, as shown below:



Since **ascending** order means **smaller to greater.**

∴ The given **numbers in ascending order**
= **–7, –4, –2, 0, 3, 5 and 6** **(Ans.)**

Symbol '<' means 'is less than' and symbol '>' means 'is greater than.'

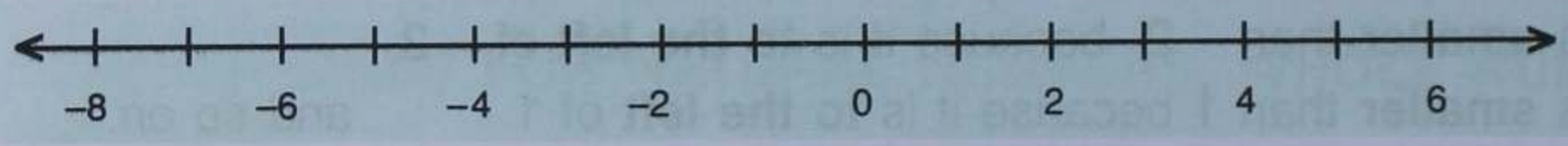
∴ Answer to Example 1 given above can also be written as :
$-7 < -4 < -2 < 0 < 3 < 5 < 6$.

If required, the same numbers in descending (decreasing) order will be written as :
6, 5, 3, 0, –2, –4 and –7

or, $6 > 5 > 3 > 0 > -2 > -4 > -7$

EXERCISE 3(A)

1. Fill in the blanks, using the following number line:



(i) An integer, on the given number line, is ..... than every number on its left.

(ii) An integer on the given number line is greater than every number to its .....

(iii) 2 is greater than –4 implies 2 is to the ..... of –4.

(iv) –3 is ..... than 2 and 3 is ..... than –2.

(v) –4 is ..... than –8 and 4 is ..... than 8.

(vi) 5 is ..... than 2 and –5 is ..... than –2.

(vii) –6 is ..... than 3 and the opposite of – 6 is ..... than opposite of 3.

(viii) 8 is ..... than –5 and –8 is ..... than –5.

2. In each of the following pairs, state *which* integer *is greater* :
(i) – 15, –23 (ii) –12, 15 (iii) 0, 8 (iv) 0, – 3

3. In each of the following pairs, state *which* integer *is smaller* :
(i) 0, – 6 (ii) 2, –3 (iii) 15, –51 (iv) 13, 0

4. In each of the following pairs, replace * with < or > to make the statement true :
(i) 3 * 0 (ii) 0 * – 8 (iii) –9 * –3 (iv) –3 * 3
(v) 5 * –1 (vi) –13 * 0 (vii) –8 * –18

5. In each case, **arrange** the given integers **in ascending order** using a number line :

   (i) –8, 0, –5, 5, 4, –1 (ii) 3, –3, 4, –7, 0, –6, 2

6. In each case, **arrange** the given integers **in descending order** using a number line :

   (i) –5, –3, 8, 15, 0, –2 (ii) 12, 23, –11, 0, 7, 6

7. For each of the statements given below, state whether it is **true** or **false** :

   (i) The smallest integer is 0. (ii) The opposite of –17 is 17.

   (iii) The opposite of zero is zero. (iv) Every negative integer is smaller than 0.

   (v) 0 is greater than every positive integer.

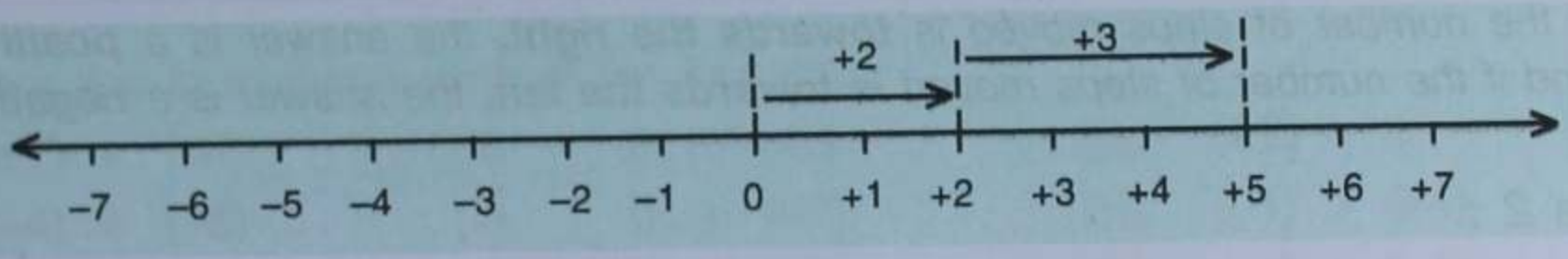
   (vi) Since zero is neither negative nor positive, it is not an integer.

3.4 USING A NUMBER LINE

A number line can be used for addition and subtraction of numbers.

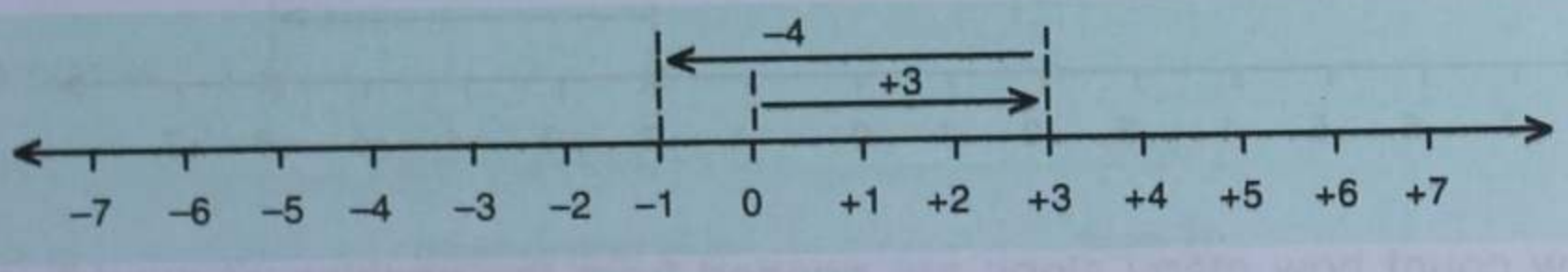
A. *For addition of numbers* :

1. *Addition of a + ve number to a + ve number,* e.g. (+ 2) + (+ 3)



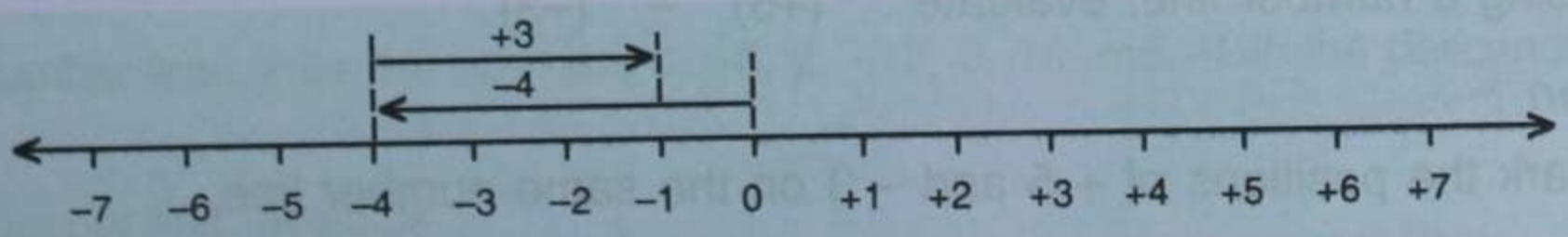
First of all, for + 2, count 2 units to the right of zero (because the right side is for the positive sign). Then for + 3, move three units to the right of + 2. You reach + 5.
Therefore, **(+ 2) + (+ 3) = + 5** or simply **5.**

2. *Addition of a + ve number to a – ve number,* e.g. (+ 3) + (– 4)



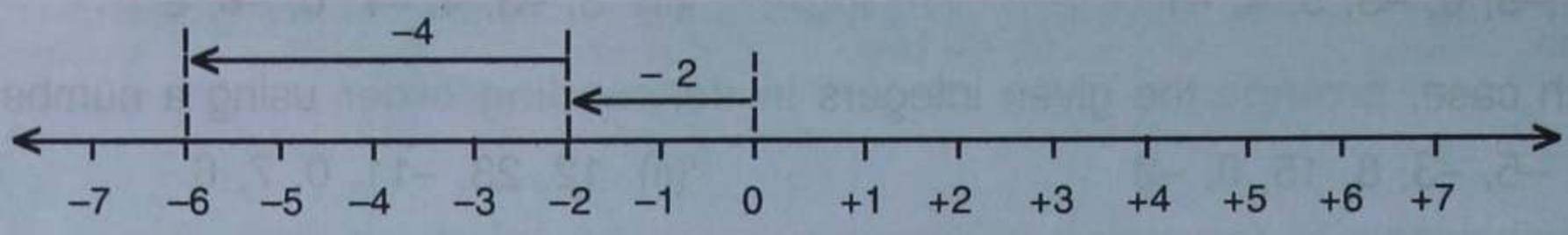
For + 3, move 3 units to the right of zero, and then, for – 4, move 4 units to the left of +3. You reach – 1.
Therefore, **(+ 3) + (– 4) = – 1.**

3. *Addition of a – ve number to a + ve number,* e.g. (– 4) + (+ 3)



For – 4, move 4 units to the left of zero, and then, for + 3, move 3 units to the right of – 4. You reach – 1.
Therefore, **(– 4) + (+ 3) = – 1.**

4. *Addition of a – ve number to a – ve number,* e.g. (– 2) + (– 4)



For – 2, start from zero and move two units to the left, and then, for – 4, move 4 units to the left of – 2. You reach – 6.

Therefore, **(– 2) + (– 4) = – 6.**

B. *For subtraction of numbers* :

To subtract a given number from another number :

1. *Mark the two given numbers on the same number line,* each starting from zero.
2. *Find out how many steps are needed to reach the position of the first number, from the position of the second number,* (*i.e.* the one which is to be subtracted).

   This number of steps is the required answer.

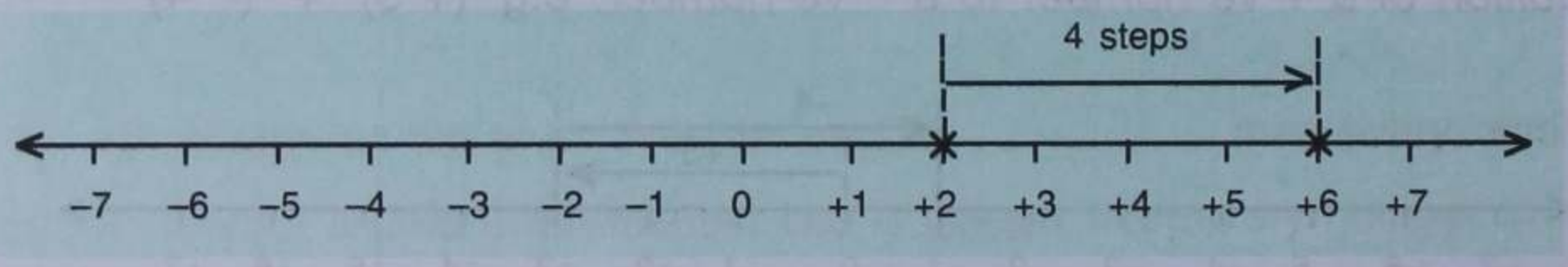
> *If the number of steps moved is* ***towards the right,*** *the answer is a* ***positive number,*** *and if the number of steps moved is* ***towards the left,*** *the answer is a* ***negative number.***

Example 2 :

Using a number line, evaluate : (+ 6) – (+ 2).

Solution :

Mark the positions of the numbers + 6 and + 2 on the same number line.



Now count how many steps are needed from the position of number + 2 to reach the position of number + 6. We find that 4 steps are needed towards right.

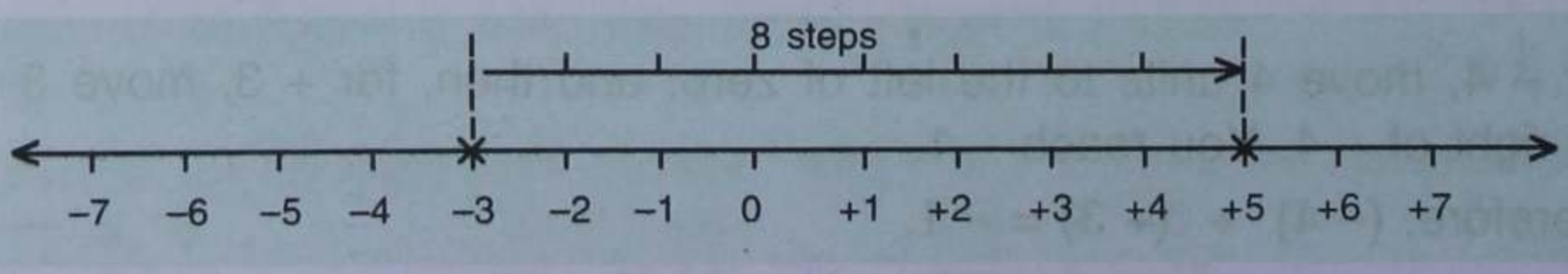
∴ **(+ 6) – (+ 2) = + 4** **(Ans.)**

Example 3 :

Using a number line, evaluate : (+5) – (–3).

Solution :

Mark the positions of + 5 and – 3 on the same number line.



Now, starting from the position of – 3, count the number of steps needed to reach + 5. Also, see the direction. We find that we have to move 8 steps to the right.

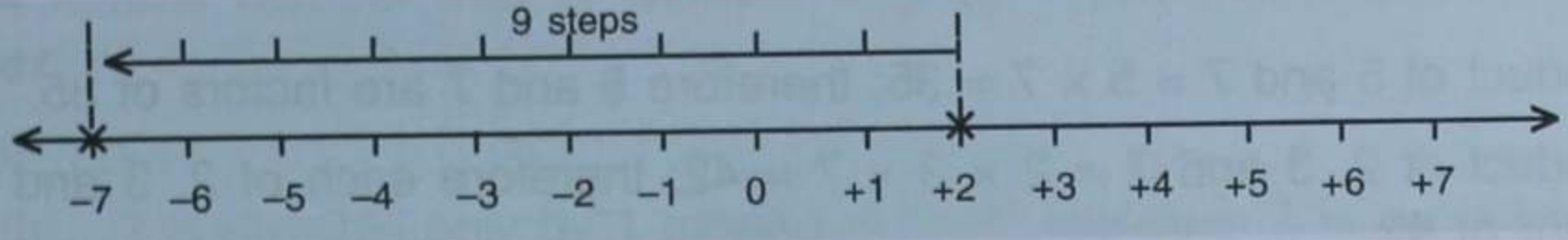
$\therefore \quad (+5) - (-3) = +8$ (Ans.)

Example 4 :

Using a number line, evaluate : $(-7) - (+2)$.

Solution :

After marking the positions of – 7 and + 2 on the same number line, count from the position of + 2 both the number of steps and the direction needed to reach – 7.



We find that we have moved 9 steps to the left.

$\therefore \quad (-7) - (+2) = -9$ (Ans.)

EXERCISE 3(B)

Use a number line to evaluate each of the following :

1.	(i)	$(+7) + (+4)$	(ii)	$0 + (+6)$	(iii)	$(+5) + 0$
2.	(i)	$(-4) + (+5)$	(ii)	$0 + (-2)$	(iii)	$(-1) + (+4)$
3.	(i)	$(+4) + (-2)$	(ii)	$(+3) + (-6)$	(iii)	$0 + (-4)$
4.	(i)	$(-1) + (-2)$	(ii)	$(-3) + (-4)$	(iii)	$(-2) + (-5)$
5.	(i)	$(+10) - (+2)$	(ii)	$(+8) - (-5)$	(iii)	$(-6) - (+2)$
	(iv)	$(-7) - (+5)$	(v)	$(+4) - (-2)$	(vi)	$(-8) - (-4)$

Revision Exercise (Chapter 3)

1. Fill in the blanks :
   (i) 5 is ..... than – 2 and –5 is ..... than 2.
   (ii) – 3 is ..... than 0 and 3 is ..... than 0.
   (iii) On a number line, if x is to the left of y, then x is ..... than y.
   (iv) On a number line, if x is to the right of y, then y is ..... than x.
2. Using a number line, write the numbers –15, 7, 0, –8 and –3 in the ascending order of value.
3. Using a number line, write the numbers 8, –6, 2, –12, 0, 15 and –1 in the descending order of value.
4. Using a number line, evaluate :

(i)	$(+5) + (+4)$	(ii)	$(+6) + (+8)$	(iii)	$(-3) + (+5)$
(iv)	$(-3) + (+7)$	(v)	$(+6) + (-2)$	(vi)	$(-3) + (+3)$
(vii)	$(-5) + (-5)$	(viii)	$(-7) + (-1)$	(ix)	$(+6) - (+2)$
(x)	$(+5) - (-3)$	(xi)	$(+4) - (-1)$	(xii)	$(-7) - (-2)$