

## POWERS AND ROOTS

## 7.1 EXPONENTIAL NOTATIONS (Positive powers only)

If number 3 is multiplied to itself 4 times, we get :  $3 \times 3 \times 3 \times 3$ .

The product of  $3 \times 3 \times 3 \times 3$  can be written as  $3^4$ .

$3^4$  is called the exponential notation for the product of  $3 \times 3 \times 3 \times 3$ .

In the product  $3 \times 3 \times 3 \times 3 = 3^4$ , the repeating number **3** is called the **base**, whereas **4** is called the **exponent** (or *index* or *power*).

Similarly : (i) in  $2^3$ , base = 2 and exponent (power) = 3

(ii) in  $(-3)^4$ , base =  $-3$  and exponent (power) = 4

(iii) in  $x^{\frac{2}{3}}$ , base =  $x$  and exponent (index) =  $\frac{2}{3}$  and so on.

In general,  $3^4$  is read as: **3 raised to the power 4** or **the fourth power of three**.

$$1. \quad (-3)^4 = -3 \times -3 \times -3 \times -3 = 81 \quad \text{and} \quad -(3)^4 = -(3 \times 3 \times 3 \times 3) = -81$$

$$2. \quad \text{For any non-zero number } a, \quad a^0 = 1$$

$$\text{i.e.} \quad 2^0 = 1, \quad (-5)^0 = 1, \quad \left(\frac{2}{3}\right)^0 = 1 \quad \text{and so on.}$$

$$3. \quad \text{For any number } a, \quad a^1 = a$$

$$\text{i.e.} \quad (-2)^1 = -2, \quad 5^1 = 5, \quad \left(\frac{2}{3}\right)^1 = \frac{2}{3} \quad \text{and so on.}$$

## Example 1 :

Evaluate : (i)  $3^2 \times 2^4$

(ii)  $3^2 \times 2^0$

## Solution :

$$(i) \quad 3^2 \times 2^4 = 3 \times 3 \times 2 \times 2 \times 2 \times 2 = 144$$

(Ans.)

$$(ii) \quad 3^2 \times 2^0 = 3 \times 3 \times 2^0 = 9 \times 1 = 9$$

(Ans.)

## Example 2 :

Simplify and express the result in exponential form :

$$(i) \quad \frac{5 \times 2 \times 3^3}{3 \times 3 \times 2 \times 5^3}$$

$$(ii) \quad \frac{3^4 \times 2^4}{2^3 \times 3^6}$$

## Solution :

$$(i) \quad \frac{5 \times 2 \times 3^3}{3 \times 3 \times 2 \times 5^3} = \frac{5 \times 2 \times 3 \times 3 \times 3}{3 \times 3 \times 2 \times 5 \times 5 \times 5} = \frac{3}{5 \times 5} = \frac{3}{5^2}$$

(Ans.)

$$(ii) \frac{3^4 \times 2^4}{2^3 \times 3^6} = \frac{3 \times 3 \times 3 \times 3 \times 2 \times 2 \times 2 \times 2}{2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3} = \frac{2}{3 \times 3} = \frac{2}{3^2} \quad (\text{Ans.})$$

**Example 3 :**Evaluate :  $6a^2b^3$  for  $a = 3$  and  $b = 2$ .**Solution :**

$$\begin{aligned} 6a^2b^3 &= 6 \times a \times a \times b \times b \times b \\ &= 6 \times 3 \times 3 \times 2 \times 2 \times 2 = 432 \end{aligned} \quad (\text{Ans.})$$

The following table will make it more clear :

Number	As a product of prime factors	In the exponential form
8	$2 \times 2 \times 2$	$2^3$
243	$3 \times 3 \times 3 \times 3 \times 3$	$3^5$
72	$2 \times 2 \times 2 \times 3 \times 3$	$2^3 \times 3^2$
630	$2 \times 3 \times 3 \times 5 \times 7$	$2^1 \times 3^2 \times 5^1 \times 7^1$
9000	$2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5 \times 5$	$2^3 \times 3^2 \times 5^3$

**EXERCISE 7(A)**

1. Fill in the blanks :

- (i) In expression  $x^y$ , base = ..... and exponent = .....  
 (ii) In expression  $(-6)^4$ , power = ..... and base = .....  
 (iii) If base = 8 and exponent = 5, then expression = .....  
 (iv) If base = -2 and power = 10, then expression = .....

2. Evaluate the following :

- (i)  $5^0 = \dots\dots\dots$  (ii)  $-5^0 = \dots\dots\dots$   
 (iii)  $-4^3 = \dots\dots\dots$  (iv)  $(-1)^7 = \dots\dots\dots$   
 (v) If  $x = 1$ , then  $15x^7 = \dots\dots\dots$  (vi) If  $y = 3$ , then  $y^3 = \dots\dots\dots$   
 (vii) If  $a = -2$ , then  $4a^3 = \dots\dots\dots$  (viii) If  $3^a = 1$ , then  $a = \dots\dots\dots$

3. Find the value of :

- (i)  $(-2)^5$  (ii)  $(-10)^3$  (iii)  $10^4$  (iv)  $7^4$

4. Find the value of :

- (i)  $\left(\frac{2}{3}\right)^4$  (ii)  $\left(-\frac{1}{2}\right)^6$  (iii)  $\left(-\frac{2}{7}\right)^3$  (iv)  $\left(\frac{3}{5}\right)^4$

5. Simplify and express the result in exponential notation :

- (i)  $\frac{2 \times 2 \times 2 \times 2 \times 3}{3 \times 3 \times 3 \times 2}$  (ii)  $\frac{3^5 \times 2^2}{3 \times 3 \times 2}$  (iii)  $\frac{5 \times 5 \times 5^5 \times 3^2}{3 \times 3 \times 5^2}$   
 (iv)  $\frac{2^3 \times 3^4 \times 5^2}{5^4 \times 3^2 \times 2}$  (v)  $\frac{5^6 \times 11^3}{11^5 \times 5 \times 5}$

6. Evaluate :

- (i)  $\frac{(-3)^3 \times (-2)^7}{(-2)^5}$  (ii)  $8^5 \div 8^2$  (iii)  $(-5)^7 \div 5^4$

7. Evaluate :

- (i)  $a^3$  for  $a = 3$  (ii)  $4b^3$  for  $b = 4$   
 (iii)  $2x^3$  for  $x = 5$  (iv)  $(3x)^2$  for  $x = 1$   
 (v)  $(4a)^3$  for  $a = -1$  (vi)  $2a^3b^2$  for  $a = 2$  and  $b = 3$   
 (vii)  $3x^3y^4$  for  $x = 1$  and  $y = -1$  (viii)  $ab^3$  for  $a = 2$  and  $b = -3$   
 (ix)  $a^3 + b^3$  for  $a = 1$  and  $b = 2$  (x)  $a^3 + b^3 - 3ab$  for  $a = 2$  and  $b = 1$

8. Express each of the following numbers in the exponential form of its prime factors :

- (i) 16 (ii) 81 (iii) 144 (iv) 2700  
 (v) 3000 (vi) 6075 (vii) 4500

## 7.2 PROPERTIES OF EXPONENTS

**Property 1 (Product Law) :**

$$a^m \times a^n = a^{m+n}$$

*For example :*

- (i)  $a^3 \times a^2 = a^{3+2} = a^5$  (ii)  $3^7 \times 3^3 = 3^{7+3} = 3^{10}$   
 (iii)  $(-5)^4 \times (-5)^2 = (-5)^6$  (iv)  $\left(\frac{3}{4}\right)^6 \times \left(\frac{3}{4}\right)^5 = \left(\frac{3}{4}\right)^{11}$  and so on.

**Property 2 (Quotient Law) :**

$$\frac{a^m}{a^n} = a^{m-n}, \text{ if } m > n \quad \text{and} \quad \frac{a^m}{a^n} = \frac{1}{a^{n-m}}, \text{ if } m < n \text{ and } a \neq 0.$$

*For example :*

- (i)  $\frac{a^5}{a^3} = a^{5-3} = a^2$  (ii)  $\frac{a^3}{a^5} = \frac{1}{a^{5-3}} = \frac{1}{a^2}$   
 (iii)  $\frac{3^7}{3^4} = 3^{7-4} = 3^3$  (iv)  $\frac{3^4}{3^7} = \frac{1}{3^{7-4}} = \frac{1}{3^3}$  and so on.

For both the properties discussed above, the base must be the same.

**Property 3 (Power Law) :**

$$(a^m)^n = a^{m \times n}$$

*For example :*

- (i)  $(a^5)^3 = a^{5 \times 3} = a^{15}$  (ii)  $(3^{-2})^5 = 3^{-2 \times 5} = 3^{-10}$  and so on.

Also, (i)  $(a^3 \times b^4)^5 = a^{3 \times 5} \times b^{4 \times 5} = a^{15} \times b^{20}$

- (ii)  $\left(\frac{3^2}{2^4}\right)^3 = \frac{3^{2 \times 3}}{2^{4 \times 3}} = \frac{3^6}{2^{12}}$  and so on.

$$(a \times b)^5 = a^5 \times b^5, \text{ but } (a + b)^5 \neq a^5 + b^5 \text{ and } (a - b)^5 \neq a^5 - b^5.$$

**Example 4 :**

Using the properties of exponents, evaluate : (i)  $\frac{2^3 \times 2^7}{2^6}$  (ii)  $\frac{(-2)^3 \times 2^7}{2^6}$  (iii)  $\frac{3^8 \times 4^3}{3^6 \times 4^4}$

**Solution :** (i)  $\frac{2^3 \times 2^7}{2^6} = \frac{2^{3+7}}{2^6} = \frac{2^{10}}{2^6}$   
 $= 2^{10-6} = 2^4 = 2 \times 2 \times 2 \times 2 = 16$  (Ans.)

(ii)  $\frac{(-2)^3 \times 2^7}{2^6} = \frac{-2^3 \times 2^7}{2^6}$   $[(-2)^3 = -2 \times -2 \times -2 = -2 \times 2 \times 2 = -2^3]$   
 $= \frac{-2^{10}}{2^6} = -2^{10-6} = -2^4 = -2 \times 2 \times 2 \times 2 = -16$  (Ans.)

(iii)  $\frac{3^8 \times 4^3}{3^6 \times 4^4} = \frac{3^{8-6}}{4^{4-3}} = \frac{3^2}{4^1} = \frac{3 \times 3}{4} = \frac{9}{4} = 2 \frac{1}{4}$  (Ans.)

### EXERCISE 7(B)

1. Fill in the blanks :

(i)  $x^a \times x^b = \dots\dots\dots$  (ii)  $a^3 \times a^8 = \dots\dots\dots$  (iii)  $a^5 \times b^3 = \dots\dots\dots$

(iv)  $2^3 \times 3^2 = \dots\dots\dots$  (v)  $\frac{5^7}{5^4} = \dots\dots\dots$  (vi)  $\frac{5^3}{5^5} = \dots\dots\dots$

(vii)  $\frac{8^6}{8^4} = \dots\dots\dots$  (viii)  $(a^3 \times b^4)^2 = \dots\dots\dots$  (ix)  $\left(\frac{3^0}{2^2}\right)^2 = \dots\dots\dots$

(x)  $(4^2 \times 5^0)^3 = \dots\dots\dots$

2. Using the properties of exponents, evaluate :

(i)  $\frac{3^4 \times 3^9}{3^{15}}$  (ii)  $\frac{4^6 \times 4^3}{4^5 \times 4^2}$  (iii)  $\frac{(-2)^6 \times (-2)^5}{(-2)^7}$

(iv)  $\frac{2^3 \times 5^2}{5^4}$  (v)  $\frac{5^2 \times 3^5}{3^3}$

3. Evaluate :

(i)  $\frac{(-3)^3 \times (-2)^4}{(-2)^2}$  (ii)  $\frac{(2^3 \times 5^4)^2}{2^4 \times 5^5}$  (iii)  $(-5)^4 \div 5^4$  (iv)  $(-3)^7 \div (-3)^6$

4. Evaluate :

(i)  $2^3 - 3^2 + 4^0$  (ii)  $5^2 + 2^2 - 3^3 + 8^0$

5. Evaluate :

(i)  $2x^2$ , if  $x = 3$  (ii)  $x^3y^2$ , if  $x = 2$  and  $y = 3$

(iii)  $x^2 + y^3$ , if  $x = 1$  and  $y = 2$

(iv)  $3x^2y - 2xy^2$ , if  $x = 5$  and  $y = 6$

(iv)  $3x^2y - 2xy^2 = 3 \times 5^2 \times 6 - 2 \times 5 \times 6^2$   
 $= 3 \times 25 \times 6 - 10 \times 36$   
 $= 450 - 360 = 90$

## 7.3 SQUARES

When a number is multiplied by itself, the product obtained is called the *square* of that number.

*For example :*

- (i) Since  $4 \times 4 = 16$ ,  $\therefore 16$  is square of 4, and we write :  $(4)^2 = 16$   
 (ii) Since  $-2 \times -2 = 4$ ,  $\therefore 4$  is square of  $-2$  and we write :  $(-2)^2 = 4$   
 (iii) Since  $\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$ ,  $\therefore \frac{4}{9}$  is square of  $\frac{2}{3}$  which is written as  $\left(\frac{2}{3}\right)^2 = \frac{4}{9}$   
 (iv) Since  $0.2 \times 0.2 = 0.04$   $\therefore 0.04$  is square of  $0.2$  i.e.  $(0.2)^2 = 0.04$  and so on.

Whether the number is positive or negative, its square is always positive.

e.g. (i)  $(3)^2 = 3 \times 3 = 9$

(ii)  $(-3)^2 = -3 \times -3 = 9$

(iii)  $(-5)^2 = -5 \times -5 = 25$  and so on

*More examples :*

(i) Square of  $0 = 0^2 = 0$

(ii) Square of  $5 = 5^2 = 5 \times 5 = 25$

(iii) Square of  $-\frac{2}{5} = \left(-\frac{2}{5}\right)^2 = \left(-\frac{2}{5}\right) \times \left(-\frac{2}{5}\right) = \frac{4}{25}$

(iv) Square of  $2\frac{3}{7} = \left(2\frac{3}{7}\right)^2 = \left(\frac{17}{7}\right)^2 = \frac{17}{7} \times \frac{17}{7} = \frac{289}{49} = 5\frac{44}{49}$

(v) Square of  $-3\frac{1}{2} = \left(-3\frac{1}{2}\right)^2 = \left(-\frac{7}{2}\right)^2 = \left(-\frac{7}{2}\right) \times \left(-\frac{7}{2}\right) = \frac{49}{4} = 12\frac{1}{4}$

(vi) Square of  $-2.3 = (-2.3)^2 = -2.3 \times -2.3 = 5.29$  and so on.

### EXERCISE 7(C)

- Find the squares of first five natural numbers.
- Find the squares of first six even natural numbers.
- Find the squares of first four odd natural numbers.
- Find the squares of first five prime numbers.

5. Find the squares of :

(i) 9                      (ii)  $\frac{2}{5}$                       (iii)  $1\frac{2}{7}$                       (iv)  $2\frac{3}{4}$

6. Find the squares of :

(i)  $-3$                       (ii)  $-\frac{2}{3}$                       (iii)  $-1\frac{2}{5}$                       (iv)  $-2\frac{1}{4}$

7. Find the squares of :

(i) 2.5                      (ii) 0.6                      (iii) 0.23  
 (iv) 0.02                      (v)  $-1.6$                       (vi)  $-0.8$

## 7.4 SQUARE ROOT

The *square root* of a given number, is that number which when multiplied by itself, gives the given number.

*For example :*

- (i) Square root of 16 = 4, as 4 multiplied by itself =  $4 \times 4 = 16$ .  
 (ii) Square root of 25 = 5, as 5 multiplied by itself =  $5 \times 5 = 25$  and so on.

## 7.5 SYMBOL FOR SQUARE ROOT

The square root is denoted by the radical sign  $\sqrt{\quad}$ .

*For example :*

- (i)  $\sqrt{9}$  means square root of 9;      (ii) Square root of  $\frac{16}{25} = \sqrt{\frac{16}{25}}$   
 (iii)  $\sqrt{0.16}$  means square root of 0.16 and so on.

## 7.6 METHODS OF FINDING THE SQUARE ROOT

(a) **To find the square root of a number (whole number) using the prime factor method :**

**Steps :**

- Express the given number as the product of its prime factors.
- Make groups, each consisting of two identical factors obtained in step (1).
- Take one factor from each group and multiply them together.
- The product so obtained is the square root of the given number.

**Example 5 :**

Find the square root of :    (i) 144      (ii) 225      (iii) 900

**Solution :**

(i) Since  $144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$

$$\therefore \text{Square root of 144} = \sqrt{144}$$

$$= \sqrt{(2 \times 2) \times (2 \times 2) \times (3 \times 3)}$$

$$= 2 \times 2 \times 3 \quad \text{[Taking one factor from each pair]}$$

$$= 12 \quad \text{(Ans.)}$$

OR, directly, as  $144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$

$$= (2 \times 2) \times (2 \times 2) \times (3 \times 3)$$

$$\therefore \text{Square root of 144} = 2 \times 2 \times 3 = 12 \quad \text{(Ans.)}$$

(ii)  $\therefore 225 = 3 \times 3 \times 5 \times 5$

$$= (3 \times 3) \times (5 \times 5)$$

$$\therefore \text{Square root of 225} = 3 \times 5 = 15 \quad \text{(Ans.)}$$

(iii)  $= \sqrt{2 \times 2 \times 3 \times 3 \times 5 \times 5}$

$$\sqrt{900} = \sqrt{2 \times 2 \times 3 \times 3 \times 5 \times 5} = 2 \times 3 \times 5 = 30 \quad \text{(Ans.)}$$

**(b) To find the square root of a number in the fraction form :**

The square root of a fraction is found by getting the square roots of its numerator and denominator separately.

$$\text{Square root of a fraction} = \frac{\text{Square root of its numerator}}{\text{Square root of its denominator}} \quad \text{i.e.} \quad \sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

For example :

$$(i) \quad \sqrt{\frac{225}{49}} = \sqrt{\frac{3 \times 3 \times 5 \times 5}{7 \times 7}} = \frac{3 \times 5}{7} = \frac{15}{7} = 2\frac{1}{7}$$

$$(ii) \quad \sqrt{4.41} = \sqrt{\frac{441}{100}} = \sqrt{\frac{3 \times 3 \times 7 \times 7}{2 \times 2 \times 5 \times 5}} = \frac{3 \times 7}{2 \times 5} = \frac{21}{10} = 2.1$$

**(c) To find the square root of a number in index form :**

In general, the square root of a given number in its index form can be obtained only when its index (power) is an even number *i.e.* the given number is of the form  $2^{10}$ ,  $3^8$ ,  $5^4$ , etc.

**Method :** Keeping the base same, divide index (power) by 2.

**Example 6 :**

Find the square root of : (i)  $2^8$       (ii)  $3^6$       (iii)  $2^4 \times 5^2$

**Solution :**

(i)  $2^8$  shows that the power of 2 is 8, and half of 8 is 4.

$$\therefore \text{Square root of } 2^8 = 2^4 = 2 \times 2 \times 2 \times 2 = 16 \quad (\text{Ans.})$$

$$\text{OR, directly, } \sqrt{2^8} = 2^4 = 2 \times 2 \times 2 \times 2 = 16 \quad (\text{Ans.})$$

Similarly :

$$(ii) \quad \sqrt{3^6} = 3^3 = 3 \times 3 \times 3 = 27 \quad (\text{Ans.})$$

$$(iii) \quad \sqrt{2^4 \times 5^2} = 2^2 \times 5^1 = 2 \times 2 \times 5 = 20 \quad (\text{Ans.})$$

More examples :

$$(i) \quad \begin{aligned} \sqrt{44 \times 22 \times 8} &= \sqrt{(2 \times 2 \times 11) \times (2 \times 11) \times (2 \times 2 \times 2)} \\ &= \sqrt{(2 \times 2) \times (11 \times 11) \times (2 \times 2) \times (2 \times 2)} \\ &= 2 \times 11 \times 2 \times 2 = 88 \end{aligned}$$

$$(ii) \quad \sqrt{\frac{3^2 \times 36}{5^4}} = \sqrt{\frac{3^2 \times (2 \times 2) \times (3 \times 3)}{5^4}} = \frac{3^1 \times 2 \times 3}{5^2} = \frac{18}{25} \quad \text{and so on.}$$

**EXERCISE 7(D)**

1. Fill in the blanks :

(i) The square of 7 is  $(7)^a$ , then  $a = \dots\dots\dots$

(ii) The square of  $b$  is 49, then  $b = \dots\dots\dots$

(iii) If  $x$  is positive and  $x^2 = 121$ , then  $x = \dots\dots\dots$

(iv) If the square of  $0.8 = 0.64$ , then the square root of  $0.64 = \dots\dots\dots$

2. Find the square root of :

- |         |          |           |            |
|---------|----------|-----------|------------|
| (i) 64  | (ii) 144 | (iii) 225 | (iv) 324   |
| (v) 441 | (vi) 484 | (vii) 625 | (viii) 729 |

3. Find the square root of :

- |                     |                     |                       |
|---------------------|---------------------|-----------------------|
| (i) $\frac{4}{9}$   | (ii) $\frac{9}{16}$ | (iii) $\frac{25}{64}$ |
| (iv) $1\frac{7}{9}$ | (v) $1\frac{9}{16}$ | (vi) $2\frac{2}{49}$  |

4. Find the square root of :

- |                              |                              |                                  |
|------------------------------|------------------------------|----------------------------------|
| (i) $4 \times 49$            | (ii) $9 \times 81$           | (iii) $100 \times 16$            |
| (iv) $1 \times 25 \times 36$ | (v) $36 \times 64 \times 81$ | (vi) $\frac{81 \times 121}{225}$ |

5. Find the square root of :

- |                   |                     |                                |
|-------------------|---------------------|--------------------------------|
| (i) $98 \times 8$ | (ii) $72 \times 18$ | (iii) $8 \times 25 \times 200$ |
|-------------------|---------------------|--------------------------------|

6. Evaluate :

- |  |  |  |
|--|--|--|
| (i) $\sqrt{144 \times 4}$                | (ii) $\sqrt{25 \times 400}$                  | (iii) $\sqrt{100 \times 36}$             |
| (iv) $\sqrt{9 \times 81 \times 100}$     | (v) $\sqrt{16 \times 25 \times 4 \times 64}$ | (vi) $\sqrt{81 \times 64 \times 4}$      |
| (vii) $\sqrt{2^2 \times 4^2 \times 3^2}$ | (viii) $\sqrt{3^6 \times 5^4 \times 2^8}$    | (ix) $\sqrt{\frac{196 \times 100}{400}}$ |
| (x) $\sqrt{\frac{2^8 \times 3^2}{5^2}}$  | (xi) $\sqrt{\frac{1^2 \times 25}{4^2}}$      | (xii) $\sqrt{\frac{81 \times 225}{100}}$ |

7. Evaluate :

- |                                  |  |  |
|----------------------------------|--|--|
| (i) $\sqrt{25} + \sqrt{16}$      | (ii) $\sqrt{49} - \sqrt{36}$           | (iii) $\sqrt{25} \times \sqrt{16}$                         |
| (iv) $\sqrt{144} \div \sqrt{16}$ | (v) $\sqrt{49} - \sqrt{16} + \sqrt{4}$ | (vi) $\frac{\sqrt{81} + \sqrt{9}}{\sqrt{196} - \sqrt{64}}$ |

## 7.7 CUBES

When a **number** is **multiplied** by itself **three times**, the product obtained is called the **cube** of that number.

*For example :*

- (i) Since  $4 \times 4 \times 4 = 64$ ,  $\therefore 64$  is the cube of 4, and we write :  $(4)^3 = 64$
- (ii) Since  $-2 \times -2 \times -2 = -8$ ,  $\therefore -8$  is the cube of  $-2$ , and we write :  $(-2)^3 = -8$
- (iii) Since  $\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{8}{27}$ ,  $\therefore \frac{8}{27}$  is the cube of  $\frac{2}{3}$ , and we write :  $\left(\frac{2}{3}\right)^3 = \frac{8}{27}$
- (iv) Since  $0.2 \times 0.2 \times 0.2 = 0.008$ ,  $\therefore 0.008$  is cube of 0.2, and we write :  $(0.2)^3 = 0.008$

1. The cube of a positive number is always positive.  
e.g.,  $(3)^3 = 3 \times 3 \times 3 = 27$ ,  $(5)^3 = 5 \times 5 \times 5 = 125$  and so on.
2. The cube of a negative number is always negative.  
e.g.,  $(-3)^3 = -3 \times -3 \times -3 = -27$ ,  $(-5)^3 = -5 \times -5 \times -5 = -125$  and so on.
3.  $-a^3 = (-a)^3$   
i.e.,  $-2^3 = (-2)^3$ ,  $-5^3 = (-5)^3$  and so on.

*More examples :*

- (i) The cube of 0 =  $0^3 = 0 \times 0 \times 0 = 0$
- (ii) The cube of 6 =  $6^3 = 6 \times 6 \times 6 = 216$
- (iii) The cube of  $-\frac{2}{3} = \left(-\frac{2}{3}\right)^3 = -\frac{2}{3} \times -\frac{2}{3} \times -\frac{2}{3} = -\frac{8}{27}$
- (iv) The cube of  $2\frac{2}{3} = \left(\frac{8}{3}\right)^3 = \frac{8}{3} \times \frac{8}{3} \times \frac{8}{3} = \frac{512}{27} = 18\frac{26}{27}$
- (v) The cube of  $-3\frac{1}{2} = \left(-3\frac{1}{2}\right)^3 = \left(-\frac{7}{2}\right)^3 = -\frac{7}{2} \times -\frac{7}{2} \times -\frac{7}{2} = -\frac{343}{8} = -42\frac{7}{8}$
- (vi) The cube of 1.2 =  $(1.2)^3 = 1.2 \times 1.2 \times 1.2 = 1.728$  and so on.

### EXERCISE 7(E)

1. Find the cubes of the first three whole numbers.
2. Find the cubes of each natural number between 3 and 8
3. Find the cubes of each integer between -3 and 3.
4. Find the cubes of each integer from -5 to -2.
5. Find the cubes of the even natural numbers between 4 and 10.
6. Find the cubes of the odd natural numbers from 3 to 9.
7. Find the cubes of :
 

(i) $\frac{4}{5}$	(ii) $2\frac{3}{4}$	(iii) $2\frac{1}{2}$	(iv) $3\frac{1}{3}$
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8. Find the cubes of :
 

(i) $-\frac{2}{5}$	(ii) $-1\frac{2}{5}$	(iii) $-2\frac{1}{4}$	(iv) $-3\frac{1}{3}$
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9. Find the cubes of :
 

(i) 0.1	(ii) 0.5	(iii) 1.5	(iv) 0.02
(v) -0.6	(vi) -0.08	(vii) -0.14	(viii) -1.6

## 7.8 CUBE ROOT

The *cube root* of a given number, is that number which when multiplied by itself three times, gives the given number.

*For example :*

- (i) The cube root of 64 = 4, as  $4 \times 4 \times 4 = 64$
- (ii) The cube root of 125 = 5, as  $5 \times 5 \times 5 = 125$

- (iii) The cube root of  $\frac{8}{27} = \frac{2}{3}$ , as  $\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{8}{27}$
- (iv) The cube root of  $-\frac{1}{8} = -\frac{1}{2}$ , as  $-\frac{1}{2} \times -\frac{1}{2} \times -\frac{1}{2} = -\frac{1}{8}$
- (v) The cube root of  $0.027 = 0.3$ , as  $0.3 \times 0.3 \times 0.3 = 0.027$  and so on.

## 7.9 SYMBOL FOR THE CUBE ROOT

The symbol for cube root is  $\sqrt[3]{\quad}$ .

That is, the cube root of 243 is written as :  $\sqrt[3]{243}$  or  $(243)^{\frac{1}{3}}$

Thus :

- (i) The cube root of  $64 = 4 \Rightarrow \sqrt[3]{64} = 4$
- (ii) The cube root of  $\frac{8}{27} = \frac{2}{3} \Rightarrow \sqrt[3]{\frac{8}{27}} = \frac{2}{3}$
- (iii) The cube root of  $-\frac{1}{8} = -\frac{1}{2} \Rightarrow \sqrt[3]{-\frac{1}{8}} = -\frac{1}{2}$
- (iv) The cube root of  $0.027 = 0.3 \Rightarrow \sqrt[3]{0.027} = 0.3$  and so on.

## 7.10 METHODS OF FINDING THE CUBE ROOT

(a) **To find the cube root of a number (integer) using the factor method :**

**Steps :**

- Express the given number as the product of its prime factors.
- Make groups [out of the prime factors obtained in step (1)], each consisting of three identical factors.
- Take one factor from each group and multiply them together.
- The product so obtained is the cube root of the given number.

**Example 7 :**

Find the cube root of : (i) 64 (ii) 216 (iii) 729

**Solution :**

The cube root of a positive number is always positive.

(i) Since,  $64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$$\therefore \text{The cube root of } 64 = \sqrt[3]{64}$$

$$= \sqrt[3]{2 \times 2 \times 2 \times 2 \times 2 \times 2}$$

$$= \sqrt[3]{(2 \times 2 \times 2) \times (2 \times 2 \times 2)}$$

$$= 2 \times 2 = 4 \quad (\text{Ans.})$$

**OR,**

Since,  $64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$$= (2 \times 2 \times 2) \times (2 \times 2 \times 2)$$

$$\therefore \text{The cube root of } 64 = 2 \times 2 = 4 \quad (\text{Ans.})$$

$$(ii) \quad \begin{aligned} \text{Since, } 216 &= 2 \times 2 \times 2 \times 3 \times 3 \times 3 \\ &= (2 \times 2 \times 2) \times (3 \times 3 \times 3) \\ \therefore \quad \text{The cube root of } 216 &= 2 \times 3 = 6 \end{aligned} \quad (\text{Ans.})$$

$$(iii) \quad \begin{aligned} \text{Since, } 729 &= 3 \times 3 \times 3 \times 3 \times 3 \times 3 \\ &= (3 \times 3 \times 3) \times (3 \times 3 \times 3) \\ \therefore \quad \text{The cube root of } 729 &= 3 \times 3 = 9 \end{aligned} \quad (\text{Ans.})$$

**Example 8 :**

Find the cube root of : (i)  $-64$  (ii)  $-3375$

**Solution :**

The cube root of a negative number is always negative.

$$(i) \quad \begin{aligned} \text{Since, } -64 &= -2 \times 2 \times 2 \times 2 \times 2 \times 2 \\ &= -(2 \times 2 \times 2) \times (2 \times 2 \times 2) \\ \therefore \quad \text{The cube root of } -64 &= -2 \times 2 = -4 \end{aligned} \quad (\text{Ans.})$$

$$(ii) \quad \begin{aligned} \text{Since, } -3375 &= -5 \times 5 \times 5 \times 3 \times 3 \times 3 \\ &= -(5 \times 5 \times 5) \times (3 \times 3 \times 3) \\ \therefore \quad \text{The cube root of } -3375 &= -5 \times 3 = -15 \end{aligned} \quad (\text{Ans.})$$

(b) **To find the cube root of a number in the fraction form :**

The cube root of a fraction =  $\frac{\text{The cube root of its numerator}}{\text{The cube root of its denominator}}$

For example :

$$(i) \quad \begin{aligned} \text{The cube root of } \frac{8}{27} &= \frac{\text{Cube root of } 8}{\text{Cube root of } 27} \\ &= \frac{2}{3} \end{aligned} \quad \text{8 = 2 \times 2 \times 2 and 27 = 3 \times 3 \times 3}$$

$$\text{i.e.} \quad \sqrt[3]{\frac{8}{27}} = \frac{\sqrt[3]{8}}{\sqrt[3]{27}} = \frac{\sqrt[3]{2 \times 2 \times 2}}{\sqrt[3]{3 \times 3 \times 3}} = \frac{2}{3}$$

$$(ii) \quad \text{Since } 0.729 = \frac{729}{1000} = \frac{3 \times 3 \times 3 \times 3 \times 3 \times 3}{2 \times 2 \times 2 \times 5 \times 5 \times 5} = \frac{(3 \times 3 \times 3) \times (3 \times 3 \times 3)}{(2 \times 2 \times 2) \times (5 \times 5 \times 5)}$$

$$\therefore \quad \text{The cube root of } 0.729 = \frac{3 \times 3}{2 \times 5} = \frac{9}{10} = 0.9$$

$$(iii) \quad \text{Since } -0.027 = -\frac{27}{1000} = -\frac{3 \times 3 \times 3}{2 \times 2 \times 2 \times 5 \times 5 \times 5}$$

$$\text{The cube root of } -0.027 = -\frac{3}{2 \times 5} = -\frac{3}{10} = -0.3$$

(c) **To find the cube root of a number in index form :**

In general, the cube root of a given number, in the index form, is obtained only when its index (power) is exactly divisible by 3.

**Method :** Keeping the base same, divide the index (power) by 3.

**Example 9 :**Find the cube root of : (i)  $2^6$  (ii)  $3^9$  (iii)  $5^3$ **Solution :**

$$(i) \quad \text{The cube root of } 2^6 = 2^{6 \div 3} \\ = 2^2 = 2 \times 2 = 4 \quad (\text{Ans.})$$

$$\text{i.e.} \quad \sqrt[3]{2^6} = 2^{6 \div 3} = 2^2 = 2 \times 2 = 4 \quad (\text{Ans.})$$

$$(ii) \quad \text{The cube root of } 3^9 = 3^{9 \div 3} \\ = 3^3 = 3 \times 3 \times 3 = 27 \quad (\text{Ans.})$$

$$(iii) \quad \text{The cube root of } 5^3 = 5^{3 \div 3} \\ = 5^1 = 5 \quad (\text{Ans.})$$

**Example 10 :**Find the cube root of : (i)  $2^6 \times 3^3$  (ii)  $5^6 \times 2^9$ **Solution :**

**Method :** Divide the power (index) of each number used by 3.

$$(i) \quad \text{The cube root of } 2^6 \times 3^3 = 2^{6 \div 3} \times 3^{3 \div 3} \\ = 2^2 \times 3^1 = 2 \times 2 \times 3 = 12 \quad (\text{Ans.})$$

$$(ii) \quad \text{The cube root of } 5^6 \times 2^9 = 5^{6 \div 3} \times 2^{9 \div 3} \\ = 5^2 \times 2^3 = 5 \times 5 \times 2 \times 2 \times 2 = 200 \quad (\text{Ans.})$$

**OR, directly,**

$$(i) \quad \sqrt[3]{2^6 \times 3^3} = 2^2 \times 3 \quad \text{Dividing each power by 3} \\ = 2 \times 2 \times 3 = 12 \quad (\text{Ans.})$$

$$(ii) \quad \sqrt[3]{5^6 \times 2^9} = 5^2 \times 2^3 \\ = 5 \times 5 \times 2 \times 2 \times 2 = 200 \quad (\text{Ans.})$$

The cube root of  $a^3 b^3 = ab$  and the cube root of  $\frac{a^3}{b^3} = \frac{a}{b}$ , but the cube root of  $a^3 + b^3 \neq a + b$  and the cube root of  $a^3 - b^3 \neq a - b$ .

**More examples :**

$$(i) \quad \sqrt[3]{24 \times 45 \times 25} = \sqrt[3]{(2 \times 2 \times 2 \times 3) \times (3 \times 3 \times 5) \times (5 \times 5)} \\ = \sqrt[3]{(2 \times 2 \times 2) \times (3 \times 3 \times 3) \times (5 \times 5 \times 5)} \\ = 2 \times 3 \times 5 = 30$$

$$(ii) \quad \sqrt[3]{\frac{50 \times 36 \times 75}{1715}} = \sqrt[3]{\frac{(5 \times 5 \times 2) \times (2 \times 2 \times 3 \times 3) \times (3 \times 5 \times 5)}{5 \times 7 \times 7 \times 7}} \\ = \sqrt[3]{\frac{(5 \times 5 \times 5) \times (2 \times 2 \times 2) \times (3 \times 3 \times 3) \times 5}{5 \times (7 \times 7 \times 7)}} \\ = \frac{5 \times 2 \times 3}{7} = \frac{30}{7} = 4\frac{2}{7}$$

## EXERCISE 7(F)

- Fill in the blanks :
  - The cube of 5 is  $5^x$ , then  $x = \dots\dots\dots$
  - The cube of  $y$  is 64, then  $y = \dots\dots\dots$
  - If  $x^3 = 125$ , then  $x = \dots\dots\dots$
  - If  $a^3 = 27$ , then  $a = \dots\dots\dots$
  - If cube of 0.5 is 0.125, then cube root of 0.125 =  $\dots\dots\dots$
- Find the cube root of :
  - 1
  - 343
  - 512
  - 3375
- Find the cube root of :
  - 1
  - 125
  - 216
  - 512
- Find the cube root of :
  - $\frac{1}{8}$
  - $3\frac{3}{8}$
  - $2\frac{10}{27}$
  - $-\frac{8}{27}$
- Find the cube root of :
  - 0.125
  - 0.064
  - 0.001
  - 3.375
  - 0.008
  - 0.064
- Find the cube root of :
  - $5^6$
  - $3^{15}$
  - $2^9 \times 3^{12}$
  - $4^6 \times 3^9 \times 2^{12}$

## Revision Exercise (Chapter 7)

- Fill in the blanks :
  - $5^0 = \dots\dots\dots$
  - $(-5)^0 = \dots\dots\dots$
  - $-5^0 = \dots\dots\dots$
  - $5^1 = \dots\dots\dots$
  - $(-5)^1 = \dots\dots\dots$
  - $-5^1 = \dots\dots\dots$
  - $5^2 = \dots\dots\dots$
  - $(-5)^2 = \dots\dots\dots$
  - $-5^2 = \dots\dots\dots$
  - $5^3 = \dots\dots\dots$
  - $(-5)^3 = \dots\dots\dots$
  - $-5^3 = \dots\dots\dots$
- State **true** or **false** :
  - $2^8 =$  a positive number
  - $-2^8 =$  a positive number
  - $(-2)^8 =$  a negative number
  - $2^5 =$  a negative number
  - $-2^5 =$  a negative number
  - $(-2)^5 =$  a positive number
- Verify that :
  - $5^2 - 3^2$  and  $4^2$  are equal.
  - $10^2 - 8^2$  and  $6^2$  are equal.
  - $12^2 + 5^2$  and  $13^2$  are equal.
- Hari planted 324 plants in such a way that there were as many rows of plants as there were number of columns. Find the number of rows and columns.
- Evaluate :
  - $\sqrt{64} - \sqrt[3]{64}$
  - $\sqrt[3]{125} + \sqrt{81}$
  - $\sqrt[3]{27} - \sqrt{49} + \sqrt[3]{216}$
- Write the number whose cube root is 0.6.
  - Write the number whose cube root is -0.6.
- Evaluate :
  - $8^0 \times 3^2 + 2^3 \times 5$
  - $5^2 - 9^2 + 3^3 \times 12^0 \times 2^2$
  - $8^2 \times 6^0 + 2^3 \times 3^2$
  - $6^2 \times 10^0 - (-5)^0 + 4$
- Evaluate :
  - $\sqrt{3 + \sqrt{169}}$
  - $\sqrt{9 - \sqrt{25}}$
  - $\sqrt{7 + \sqrt{81}}$